PERTURBATIONS IN THE EINSTEIN THEORY OF GRAVITY: COSERVED CURRENTS

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General relativity in the form where gravitational perturbations together with other physical fields propagate on an auxiliary background is considered. With using the Katz-Bičák-Lynden-Bell technique new conserved currents, divergences of antisymmetric tensor densities (superpotentials), in arbitrary curved spacetime are constructed.

1   A short review and setting tasks.

Frequently, the perturbed Einstein equations are suggested as follows. The linear in metric perturbations terms are placed on the left hand side; whereas all nonlinear terms are transported to the right hand side, and together with a matter energy-momentum tensor are treated as a total (effective) energy-momentum tensor $t_{\mu\nu}^{(tot)}$. A such consideration was developed in a form of a theory of a tensor field with self-interaction in a fixed background spacetime where $t_{\mu\nu}^{(tot)}$ is obtained by variation of an action with respect to a background metric $\overline{g}^{\mu\nu}$. Sometimes it is called as a *field formulation* of general relativity (GR) (see, e.g., [1]). One of key works in the field formulation of GR is the paper by Deser [2] who generalized previous authors and, using the 1-st order formalism, suggested it in the closed form (without expansions) in a Minkowski spacetime. It is also important the work by Bičák [3], where keeping quadratic approximation on a curved Ricci flat background, he has proved the theorem: with a necessity $t_{\mu\nu}^{(tot)}$ has to contain the second derivatives of gravitational variables.

In [4] the field formulation of GR was constructed on arbitrary curved backgrounds with all properties of a field theory in a fixed spacetime. These results obtained a development and have applications at the present time. Thus, in [5], a requirement only of the first derivatives in the symmetrical energy-momentum tensor and using the notions of the work [4] have led to a *new* field formulation of GR in a Minkowski spacetime. Here, there is no any contradiction with the theorem proved in the paper [3] because in [5] a left hand side, unlike standard approaches, is not linear. In [6], on the basis of the work [4] the total energy and angular momentum for $d + 1$-dimensional asymptotically anti-de Sitter spacetime were constructed. The properties of the field approach [4] appear under consideration in many of concrete problems independently. For example, in [7] and [8] a consideration of linear perturbations

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on a Friedmann-Robertson-Walker backgrounds leads to the linear approximation of the full theory [4]. In [9], in the framework of the field approach a class of so-called 'slightly bimetric' gravitation theories was constructed, and in [10] a behaviour of light cones was examined. A more full, as we know, the modern bibliography related to the field approach in gravity can be also found in [10].

Many of problems in the modern cosmology and relativistic astrophysics are related to study of an evolution of perturbations and conservation laws on given spacetime backgrounds. Inflation models and discovery of accelerated expansion of universe [11] initiate an interest to consideration of perturbations on backgrounds of de Sitter's solutions with the cosmological constant. Due to more high exactness of observations it is necessary nowadays to consider not only linear perturbations, but next orders of them too. It is important to construct conservation laws for perturbations on the backgrounds, including (anti-)de Sitter's ones (see, e.g., [6], [12] - [15]). The problems like these are also important reasons for a development of the field formulation of GR. Because in GR it is impossible to define in an unique way such quantities like energy there are many other approaches (see recent review [16]). Among them one is more physically consequent, it is the monade method for descriptions of reference systems based on timelike word lines of observers [17, 18]. The cornerstone of the construction is the field of a monade — a unite vector tangential to a line of an observer. With its use the energy-momentum densities of matter as well of gravitational field are defined.

Return to the field approach. In spite of all the successes some questions were unresolved up to now. First, for constructing currents, as usual, $t^{(tot)\nu}_\mu$ is contracted with Killing vectors $\overline{\xi}^\mu$ of the background [4]: $\hat{i}^\nu = \sqrt{-\overline{g}}t^{(tot)\nu}_\mu\overline{\xi}^\mu$. The current is differentially conserved $\hat{i}^\nu{}_{,\nu} = \hat{i}^\nu{}_{,\nu} = 0$ if the covariant differential conservation law $t^{(tot)\nu}_\mu{}_{;\nu} = 0$ holds. The last has a place only on flat, Ricci flat and (anti-)de Sitter backgrounds, whereas for more complicated backgrounds (like many of cosmological ones) $t^{(tot)\nu}_\mu{}_{;\nu} \neq 0$. This fact was interpreted [4] *qualitatively only*: as an interaction between dynamical and background systems. However, it is necessary to give the exact mathematical description, and we note this as a problem (A). Second, superpotentials (antisymmetric tensor densities, divergences of which are conserved currents) in GR play very important role (see [13, 19] and references there in). However, the construction of superpotentials in the framework of the field formulation of GR was not developed enough. Note it as a problem (B), concerning which we could tell only the following. The linear left hand side of the field equations is divergenceless (at least on flat backgrounds), therefore it is already expressed through a superpotential. Basing on this Abbott and Deser [12] have constructed a superpotential with the Killing vectors on the (anti-)de Sitter backgrounds.

In the present paper we resolve the problems (A) and (B) and show that they are connected. In the result, in the framework of the field approach we construct new superpotentials and corresponding currents on arbitrary curved backgrounds and with arbitrary displace-

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ment vectors.

2 The field formulation of GR.

Let us present the main properties of the description in [4] in the 2-nd order formalism. Consider a so-called dynamical Lagrangian [20, 21]:

$$\hat{\mathcal{L}}^{dyn} = \hat{\mathcal{L}}^E_{(dec)} - \hat{l}^{\mu\nu}\frac{\delta\overline{\hat{\mathcal{L}}^E}}{\delta\overline{\hat{g}^{\mu\nu}}} - \phi^A\frac{\delta\overline{\hat{\mathcal{L}}^E}}{\delta\overline{\Phi^A}} - \overline{\hat{\mathcal{L}}^E} - \frac{1}{2\kappa}\hat{k}^\alpha{}_{,\alpha},\tag{1}$$

the construction of which is based on the usual Lagrangian of GR

$$\hat{\mathcal{L}}^E = -(2\kappa)^{-1}\hat{R}(g_{\mu\nu}) + \hat{\mathcal{L}}^M(\Phi^A, g_{\mu\nu})\tag{2}$$

with the metric $g_{\mu\nu}$, scalar curvature $R = g^{\alpha\beta}R_{\alpha\beta}$ and the matter variables Φ^A, which are a set of arbitrary tensor densities (not spinors). Particular derivatives are $(,_\alpha)$; the hats " ˆ " mean densities of the weight $+1$; $\delta/\delta a$ is a Lagrangain derivative; $\hat{\mathcal{L}}^E_{(dec)}$ means the Lagrangian (2) after substitution of the decompositions

$$\hat{g}^{\mu\nu} \equiv \overline{\hat{g}^{\mu\nu}} + \hat{l}^{\mu\nu}, \qquad \Phi^A \equiv \overline{\Phi^A} + \phi^A.\tag{3}$$

The perturbations $\hat{l}^{\mu\nu}$ and ϕ^A are independent gravitational and matter dynamic variables; bars mean background quantities which are given, and $\overline{\hat{g}^{\mu\nu}}$ and $\overline{\Phi^A}$ satisfy the background Einstein and matter equations; indeces are shifted by the background metric. We set also

$$\hat{k}^\mu = (\overline{\hat{g}^{\mu\rho}} + \hat{l}^{\mu\rho})(\Gamma^\sigma_{\rho\sigma} - \overline{\Gamma}^\sigma_{\rho\sigma}) - (\overline{\hat{g}^{\rho\sigma}} + \hat{l}^{\rho\sigma})(\Gamma^\mu_{\rho\sigma} - \overline{\Gamma}^\mu_{\rho\sigma})\tag{4}$$

where $\Gamma^\alpha_{\mu\nu}$ are the Christoffel symbols depending on $\hat{g}^{\mu\nu}$ as on the sum in (3).

It is useful to present the Lagrangian (1) as a sum of the pure gravitational and the matter parts: $\hat{\mathcal{L}}^{dyn} = -(2\kappa)^{-1}\hat{\mathcal{L}}^g + \hat{\mathcal{L}}^m$. As is seen, the Lagrangian (1) is obtained from $\hat{\mathcal{L}}^E_{(dec)}$ subtracting zero's and linear's in $\hat{l}^{\mu\nu}$ and in ϕ^A terms of the functional expansion of $\hat{\mathcal{L}}^E_{(dec)}$. Zero's term is the background Lagrangian, and the linear term is proportional to the operators of the background equations. The variation of $\hat{\mathcal{L}}^{dyn}$ with respect to $\hat{l}^{\mu\nu}$ and ϕ^A, some algebraic transformations, and taking into account the background equations give the Einstein gravitational equations in the form

$$\hat{G}^L_{\mu\nu} + \hat{\Phi}^L_{\mu\nu} = \kappa\left(\hat{t}^g_{\mu\nu} + \hat{t}^m_{\mu\nu}\right) \equiv \kappa\hat{t}^{(tot)}_{\mu\nu}.\tag{5}$$

Here, the left hand side linear in $\hat{l}^{\mu\nu}$ and ϕ^A is expressed by

$$\hat{G}^L_{\mu\nu} \equiv \frac{\delta}{\delta g^{\mu\nu}}\hat{l}^{\rho\sigma}\frac{\delta\overline{\hat{R}}}{\delta\hat{g}^{\rho\sigma}} \equiv \tfrac{1}{2}\left(\hat{l}_{\mu\nu;\rho}{}^{;\rho} + \overline{g}_{\mu\nu}\hat{l}^{\rho\sigma}{}_{;\rho\sigma} - \hat{l}_\mu{}^\rho{}_{;\nu\rho} - \hat{l}_\nu{}^\rho{}_{;\mu\rho}\right),\tag{6}$$

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$$\hat{\Phi}^L_{\mu\nu} \equiv -2\kappa \frac{\delta}{\delta \overline{g^{\mu\nu}}} \left(\hat{l}^{\rho\sigma} \frac{\delta \overline{\hat{\mathcal{L}}^M}}{\delta \hat{\overline{g}}^{\rho\sigma}} + \phi^A \frac{\delta \overline{\hat{\mathcal{L}}^M}}{\delta \overline{\Phi^A}} \right) \tag{7}$$

with the covariant derivatives $(_{;\mu})$ with respect to $\overline{g}_{\mu\nu}$. The right hand side of Eq. (5) is the symmetrical (metric) total energy-momentum tensor density:

$$\hat{t}^{(tot)}_{\mu\nu} \equiv 2 \frac{\delta \hat{\mathcal{L}}^{dyn}}{\delta \overline{g^{\mu\nu}}} \equiv 2 \frac{\delta}{\delta \overline{g^{\mu\nu}}} \left(-\frac{1}{2\kappa} \hat{\mathcal{L}}^g + \hat{\mathcal{L}}^m \right) \equiv \hat{t}^g_{\mu\nu} + \hat{t}^m_{\mu\nu}. \tag{8}$$

Explicit expressions for $\hat{\mathcal{L}}^g$ and $\hat{t}^g_{\mu\nu}$ follow from the Lagrangian (1) and can be found in [4].

3   Superpotentials and conserved currents.

To construct conservation laws in the field formulation of GR we use the method developed by Katz, Bičák and Lynden-Bell (KBL) [13]. They are based on the Lagrangian:

$$\hat{\mathcal{L}}_G = -(2\kappa)^{-1} \left(\hat{R} + \hat{k}^\mu{}_{,\mu} - \overline{\hat{R}} \right) \tag{9}$$

depending on the physical metric $g_{\mu\nu}$ (without decompositions) and the background one $\overline{g}_{\mu\nu}$; $\hat{k}^\mu$ is also defined by Eq. (4), only with $\hat{l}^{\mu\nu} = \hat{g}^{\mu\nu} - \hat{\overline{g}}^{\mu\nu}$. The identity $\pounds_\xi \hat{\mathcal{L}}_G + (\xi^\mu \hat{\mathcal{L}}_G)_{,\mu} \equiv 0$, that has a place for $\hat{\mathcal{L}}_G$ as a scalar density, is transformed into the main identity of the KBL approach:

$$\left(\frac{\partial \hat{\mathcal{L}}_G}{\partial g_{\rho\sigma;\mu}} g_{\rho\sigma;\nu} - \hat{\mathcal{L}}_G \delta^\mu_\nu \right) \xi^\nu + 2 \left(\frac{\partial \hat{\mathcal{L}}_G}{\partial g_{\nu\tau;\mu}} g_{\nu(\tau} \delta^\rho_{\lambda)} \overline{g}^{\lambda\sigma} \right) \xi_{[\sigma,\rho]}$$
$$+ 2 \left(\frac{\delta \hat{\mathcal{L}}_G}{\delta g_{\rho\sigma}} g_{\rho(\sigma} \delta^\mu_{\nu)} + \frac{\delta \hat{\mathcal{L}}_G}{\delta \overline{g}_{\rho\sigma}} \overline{g}_{\rho(\sigma} \delta^\mu_{\nu)} \right) \xi^\nu + \hat{\zeta}^\mu \equiv \hat{I}^{\mu\nu}{}_{;\nu} \equiv \hat{I}^{\mu\nu}{}_{,\nu}. \tag{10}$$

Here, the Lie derivative, say, of a vector ψ^α with respect to ξ^α is defined as $\pounds_\xi \psi^\alpha \equiv -\xi^\beta \psi^\alpha{}_{;\beta} + \xi^\alpha{}_{;\beta} \psi^\alpha$. In Eq. (10), (i) the first term $(\hat{t}^\mu_\nu + (2\kappa)^{-1} \hat{l}^{\rho\sigma} \overline{R}_{\rho\sigma} \delta^\mu_\nu) \xi^\nu$ includes the canonical energy-momentum tensor density $\hat{t}^\mu_\nu$ for the free gravitational field; (ii) the second term is presented by the spin tensor density $\hat{\sigma}^{\mu\rho\sigma}$ contracted with $\xi_{[\sigma;\rho]}$; (iii) the third term is $\kappa^{-1}(\hat{G}^\mu_\nu - \overline{\hat{G}^\mu_\nu}) \xi^\nu$ with the Einstein tensor G^μ_ν; (iv) the fourth term is equal to zero if ξ^ν is a Killing vector of the background. Antisymmetric tensor density $\hat{I}^{\mu\nu}$ on the right hand side of (10) is the KBL superpotential generalizing the well known Freud superpotential [22] on arbitaray curved backgrounds. Explicitly all the expressions can be found in [13]. Using the dynamic $\hat{G}^\mu_\nu = \kappa \hat{T}^\mu_\nu$ and background $\overline{\hat{G}^\mu_\nu} = \kappa \overline{\hat{T}^\mu_\nu}$ Einstein equations, KBL transform the identity (10) into a 'weak' conservation law for the current $\hat{I}^\mu$ $(\partial_\mu \hat{I}^\mu = 0)$:

$$\left[\hat{t}^\mu_\nu + (\hat{T}^\mu_\nu - \overline{\hat{T}^\mu_\nu}) + (2\kappa)^{-1} \hat{l}^{\rho\sigma} \overline{R}_{\rho\sigma} \delta^\mu_\nu \right] \xi^\nu + \hat{\sigma}^{\mu\rho\sigma} \xi_{[\sigma,\rho]} + \hat{\zeta}^\mu \equiv \hat{I}^\mu = \hat{I}^{\mu\nu}{}_{,\nu}. \tag{11}$$

To apply the KBL technique in the field approach we use $\hat{g}^{\mu\nu} - \overline{\hat{g}^{\mu\nu}}$ instead of $\hat{l}^{\mu\nu}$. Then, the gravitational part $-(2\kappa)^{-1} \hat{\mathcal{L}}^g$ of (1) depending on the first derivatives of $\hat{l}^{\mu\nu}$ only goes

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to $\hat{\mathcal{L}}^{(2)}$ and is expressed over the KBL Lagrangian (9) as $\hat{\mathcal{L}}^{(2)} \equiv \hat{\mathcal{L}}_G - \hat{\mathcal{L}}^{(1)}$ with $\hat{\mathcal{L}}^{(1)} = -(2\kappa)^{-1}(\hat{g}^{\mu\nu} - \overline{\hat{g}}^{\mu\nu})\overline{R}_{\mu\nu}$. After that we transform the identity $\pounds_\xi \hat{\mathcal{L}}^{(2)} + (\xi^\mu \hat{\mathcal{L}}^{(2)})_{,\mu} \equiv 0$ into

$$\left(\frac{\partial \hat{\mathcal{L}}_G}{\partial g_{\rho\sigma;\mu}} g_{\rho\sigma;\nu} - \hat{\mathcal{L}}_G \delta^\mu_\nu + \hat{\mathcal{L}}^{(1)} \delta^\mu_\nu - 2 \frac{\delta \hat{\mathcal{L}}^{(1)}}{\delta g_{\rho\sigma}} g_{\rho(\sigma} \delta^\mu_{\nu)} \right) \xi^\nu - \hat{\mathcal{M}}_\lambda^{(2)\mu\rho} \overline{g}^{\sigma\lambda} \xi_{\sigma;\rho}$$
$$+ 2 \left(\frac{\delta \hat{\mathcal{L}}_G}{\delta g_{\rho\sigma}} g_{\rho(\sigma} \delta^\mu_{\nu)} + \frac{\delta \hat{\mathcal{L}}^{(2)}}{\delta \overline{g}_{\rho\sigma}} \overline{g}_{\rho(\sigma} \delta^\mu_{\nu)} \right) \xi^\nu \equiv - \left(\hat{\mathcal{M}}_\lambda^{(2)\mu\nu} \xi^\lambda \right)_{,\nu}. \tag{12}$$

Here,

$$\hat{\mathcal{M}}_\lambda^{(2)\mu\nu} \equiv -2 \left(\frac{\partial \hat{\mathcal{L}}^{(2)}}{\partial g_{\rho\sigma;\mu}} g_{\rho(\sigma} \delta^\nu_{\lambda)} + \frac{\partial \hat{\mathcal{L}}^{(2)}}{\overline{g}_{\rho\sigma,\nu}} \overline{g}_{\rho(\sigma} \delta^\nu_{\lambda)} \right) \tag{13}$$

is antisymmetric in μ and ν and expressed over the spin term presented in Eqs. (10) - (11) as

$$\hat{\mathcal{M}}_\lambda^{(2)\mu\nu} \overline{g}^{\lambda\rho} = \hat{\sigma}^{\rho[\mu\nu]} + \hat{\sigma}^{\mu[\rho\nu]} - \hat{\sigma}^{\nu[\rho\mu]}. \tag{14}$$

Noting that $\hat{G}_{\mu\nu}^L$ in (6) can be thought as depending on $\hat{g}^{\mu\nu}$ as well on $\hat{l}^{\mu\nu}$, it is important to find that in Eq. (12):

$$2 \frac{\delta \hat{\mathcal{L}}^{(2)}}{\delta \overline{g}_{\rho\sigma}} \overline{g}_{\rho(\sigma} \delta^\mu_{\nu)} \equiv -\frac{1}{\kappa} \hat{G}_\nu^{L\mu}.$$

Subtracting Eq. (12) from Eq. (10), replacing $\hat{g}^{\mu\nu}$ by $\hat{l}^{\mu\nu}$ in correspondence with (3) we obtain the identity:

$$\frac{1}{\kappa} \hat{G}_\nu^{L\mu} \xi^\nu + \frac{1}{\kappa} \hat{l}^{\mu\lambda} \overline{R}_{\lambda\nu} \xi^\nu + \hat{\zeta}_{(*)}^\mu \equiv \hat{I}_{(*);\nu}^{\mu\nu} \equiv \hat{I}_{(*),\nu}^{\mu\nu} \tag{15}$$

with the new superpotential $\hat{I}_{(*)}^{\mu\nu} = \hat{I}^{\mu\nu} + \hat{\mathcal{M}}_\lambda^{(2)\mu\nu} \overline{g}^{\lambda\rho} \xi_\rho$. The quantity $\hat{\zeta}_{(*)}^\mu = \hat{\zeta}^\mu + \hat{\mathcal{M}}_\lambda^{(2)\mu\rho} \overline{g}^{\lambda\sigma} \xi_{(\sigma;\rho)}$ is equal to zero on Killing vectors of a background, like $\hat{\zeta}^\mu$ in (10) and (11).

To obtain weak conservation laws one has to substitute Einstein's equations (5) into the identity (15). Note that the quantity $\hat{\Phi}_{\mu\nu}^L$ defined in Eq. (7) can be excluded from Eq. (5). Indeed, substituting (1) and (2) into the definition (8) one finds that the expression (7) appears also on the right hand side of equations (5), and thus they are rewritten as

$$\hat{G}_{\mu\nu}^L = \kappa \left(\hat{t}_{\mu\nu}^g + \delta \hat{t}_{\mu\nu}^M \right) \tag{16}$$

with

$$\delta \hat{t}_{\mu\nu}^M \equiv 2 \frac{\delta}{\delta \hat{g}^{\mu\nu}} \left[\hat{\mathcal{L}}^M \left(\overline{\Phi}^A + \phi^A, \, \overline{\hat{g}}^{\mu\nu} + \hat{l}^{\mu\nu} \right) - \overline{\hat{\mathcal{L}}^M} \right]. \tag{17}$$

Substituting Eq. (16) into Eq. (15) we obtain:

$$\left(\hat{t}_\nu^{g\mu} + \delta \hat{t}_\nu^{M\mu} + \kappa^{-1} \hat{l}^{\mu\lambda} \overline{R}_{\lambda\nu} \right) \xi^\nu + \hat{\zeta}_{(*)}^\mu \equiv \hat{\mathcal{T}}_{(*)\nu}^\mu \xi^\nu + \hat{\zeta}_{(*)}^\mu \equiv \hat{I}_{(*)}^\mu = \hat{I}_{(*),\nu}^{\mu\nu}. \tag{18}$$

It gives a conserved law $\partial_\mu \hat{I}_{(*)}^\mu = 0$ for the current $\hat{I}_{(*)}^\mu$ that takes a place (i) on arbitrary curved backgrounds including all cosmological solutions; (ii) for arbitrary displacement vectors ξ^μ, not only for the Killing ones. (iii) We present also the explicit term $\kappa^{-1} \hat{l}^{\mu\lambda} \overline{R}_{\lambda\nu}$ (included

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into $\hat{\mathcal{T}}^{\mu}_{(*)\nu}$), which describes the interaction with a background. (iv) Constructing Eq. (18) we suggested a new superpotential, the explicit form of which after calculations in (15) with (13) and the KBL superpotential is given by

$$\hat{I}^{\mu\nu}_{(*)} = \kappa^{-1}\hat{l}^{\rho[\mu}\xi^{\nu]}{}_{;\rho} + \kappa^{-1}\left(\overline{g}^{\rho[\mu}\hat{l}^{\nu]\sigma} - \overline{g}^{\sigma[\mu}\hat{l}^{\nu]\rho}\right)_{;\sigma}\xi_\rho, \qquad (19)$$

where the coefficient at ξ_ρ is a covariantized Papapetrou superpotential [23].

Thus, the new conservation law in the form of the expression (18) resolves the problems (A) and (B) annonced in the first section. The question arises: why we have a success with Eq. (18)? First, instead of $\hat{t}^m_{\mu\nu}$ in Eq. (5) we use the modified matter energy-momentum density $\delta\hat{t}^M_{\mu\nu}$ in Eq. (16) with the definition (17), that does not follow by the usual way from the Lagrangian (1). Second, due to the term $\kappa^{-1}\hat{l}^{\mu\lambda}\overline{R}_{\lambda\nu}$ the new energy-momentum tensor density $\hat{\mathcal{T}}^{\mu\nu}_{(*)}$ is *non-symmetrical* on general complicated backgrounds, whereas before only the *symmetrical* $\hat{t}^{(tot)}_{\mu\nu}$ was used.

Let us do remarks. 1) Currents and superpotentials in [19] obtained with applying the Belinfante method to the canonical system developed in [13]. They exactly coincide with the conserved quantities given here in (18) and (19). Thus the Belinfante procedure becomes 'a bridge' connecting two approaches for constructing conservation laws: *canonical* [13] and *symmetrical* [4] ones. 2) The field formulation of GR can be constructed with the use of the decomposition $g_a = \overline{g}_a + h_a$ of any metrical variable from the set $g_a \in g_{\mu\nu}$, $g^{\mu\nu}$, $\sqrt{-g}g_{\mu\nu}$, $\dots$ [21]. Then field equations, currents and superpotentials acquire the forms respectively of (5), (18) and (19) with the replacement of $\hat{l}^{\mu\nu}$ by $\hat{l}^{\mu\nu}_a = h_a(\partial\overline{\tilde{g}}^{\mu\nu}/\partial\overline{g}_a)$. The use of different definitions for $\hat{l}^{\mu\nu}_a$ leads to an ambiguity in the definition of energy-momentum tensor (as firstly was noted in [24]) and superpotentials beginning from the 2-nd order in perturbations. On the other hand, in [13, 19] the construction of the currents and superpotentials does not depend on a choice of a dynamical variable from the set g_a. Therefore, results [19] coinciding only with (18) and (19), resolve this ambiguity in favour of the decomposition (3).

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